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PARSEVAL'S AND MODULATION THEOREM FOR GENERALIZED TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM

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ABSTRACT

The concept of Fourier Transformation and Laplace Transformation play an important role in diverse areas of Science, Engineering and Technology. Fourier Transform and Laplace Transform is also play an important role in the analysis of all kinds of physical phenomena. As a link between the various applications of these transforms the authors use the theory of signal and systems, as well as the theory of ordinary and partial differential equations. In this paper we established a Two Dimensional Fourier-Laplace Transform and investigated the Linearity property, Parseval's theorem and Modulation theorem of Two Dimensional Fourier-Laplace Transform. The work may be useful for solving higher order ordinary and partial differential equations as well as integral equations.

KEYWORDS: Fourier Transform, Laplace Transform, Fourier-Laplace Transform, Generalized function.

1. INTRODUCTION

Mathematics is pivotal to understand the behavior and working of mechanical and electrical systems. The basic and sophisticated tools for solving these systems are differentiations and integrations. But some complexity arises in solving higher order differential equations. To overcome such complex higher order differential equations, the effective mathematical methods are Fourier transform and Laplace Transform. These transforms higher order differential equations into simple polynomial which is very easy to solve.

Laplace transforms are frequently opted for signal processing. Along with the Fourier transform, the Laplace transform is used to study signals in the frequency domain. Like the Laplace transform, Fourier transform is the simplest among the other transformation method. The Fourier Transform is greatly helped in various systems like wireless, signal processing, mechanical and industrial applications as well as for analyzing/analyzes the fault in power system.

Due to the wide applications of Fourier transform and Laplace transform we can developed a new integral transform by combining these transform we get an elegant integral transform that is Two Dimensional Fourier-Laplace Transform which will also be used in several fields.

In this paper we established some properties of Two Dimensional Fourier-Laplace transform. Modulation property is the most powerful concept in the signal processing, radar technology, pattern reorganization and many more in the integral transform. Parseval identity is also applied in the conservation of energy in the universe [1]. It is also useful in evaluating some definite integral [2].

The plan of this paper is as follows:
Definitions are given in section 2, in section 3; Linearity property of Two Dimensional Fourier-Laplace Transform is given. Parseval's Theorem for Two Dimensional Fourier-Laplace Transform is proved in section 4. Modulation property for Two Dimensional Fourier-Laplace Transform describes in section 5. Lastly conclusions are given in section 6

The notations and Terminologies are as per Zemanian [10], [11].

2. DEFINITIONS

The Two Dimensional Fourier Transform with the parameters s, u of function $f(t,z)$ denoted by $F[f(t,z)] = F(s,u)$ and is given by

$$F[f(t,z)] = F(s,u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(st+uz)} f(t,z)\,dtdz \tag{2.1}$$

The Two Dimensional Laplace Transform with the parameters p, v of function $f(x,y)$ denoted by $L[f(x,y)] = F(p,v)$ and is given by

$$L[f(x,y)] = F(p,v) = \int_0^{\infty}\int_0^{\infty} e^{-px-qy} f(x,y)\,dxdy \tag{2.2}$$

The Two Dimensional Fourier-Laplace Transform with parameters s, u, p, v of function $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} f(t,z,x,y)\, e^{-i\{(st+uz)-i(px+vy)\}} dt\,dz\,dx\,dy \tag{2.3}$$

Where the kernel $K(s,u,p,v) = e^{-i\{(st+uz)-i(px+vy)\}}$

The Two Dimensional Inverse Fourier Transform is defined as

$$f(t,z) = F^{-1}[F(s,u)] = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(st+uz)} F(s,u)\,ds\,du \tag{2.4}$$

The two dimensional Inverse Laplace transform is defined as,

$$f(x,y) = L^{-1}[F(p,v)] = -\frac{1}{4\pi^2}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{px+vy} F(p,v)\,dp\,dv \tag{2.5}$$

The Two Dimensional Inverse Fourier-Laplace Transform is defined as,

$$f(t,z,x,y) = FL^{-1}[F(s,u,p,v)] = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{i\{(st+uz)-i(px+vy)\}} F(s,u,p,v)\,ds\,du\,dp\,dv \tag{2.6}$$

3. LINEARITY PROPERTY OF TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM

If $FL\{f(t,z,x,y)\}$ is generalized two dimensional Fourier-Laplace transform of $f(t,z,x,y)$ and $FL\{g(t,z,x,y)\}$ is generalized two dimensional Fourier-Laplace transform of $g(t,z,x,y)$ then

$$FL\{C_1\, f(t,z,x,y) + C_2\, g(t,z,x,y)\}(s,u,p,v)$$
$$= C_1 FL\{\, f(t,z,x,y)\}(s,u,p,v) + C_2 FL\{\, g(t,z,x,y)\}(s,u,p,v)$$

Proof: Consider,

$$FL\{C_1\, f(t,z,x,y) + C_2\, g(t,z,x,y)\}(s,u,p,v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} [C_1\, f(t,z,x,y) + C_2\, g(t,z,x,y)]\, e^{-i\{(st+uz)-i(px+vy)\}}\, dt\,dz\,dx\,dy$$

$$= C_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y) e^{-i\{(st+uz)-i(px+vy)\}}\, dt\, dz\, dx\, dy$$

$$+ C_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} g(t,z,x,y) e^{-i\{(st+uz)-i(px+vy)\}}\, dt\, dz\, dx\, dy$$

$$= C_1 FL\{ f(t,z,x,y)\}(s,u,p,v) + C_2 FL\{ g(t,z,x,y)\}(s,u,p,v)$$

4. PARSEVAL'S THEOREM

Theorem: If $FL\{f(t,z,x,y)\} = F(s,u,p,v)$ and $FL\{g(t,z,x,y)\} = G(s,u,p,v)$ then

(i) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)\overline{g(t,z,x,y)}\, dt\, dz\, dx\, dy$$

$$= -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} F(s,u,p,v)\overline{G(s,u,p,v)}\, ds\, du\, dp\, dv$$

(ii) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \left|f(t,z,x,y)\right|^2 dt\, dz\, dx\, dy = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} \left|F(s,u,p,v)\right|^2 ds\, du\, dp\, dv$$

Proof: By definition,

$$FL\{g(t,z,x,y)\} = G(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\{(st+uz)-i(px+vy)\}} g(t,z,x,y)\, dt\, dz\, dx\, dy \quad (4.1)$$

Using the inversion formula for Two Dimensional Fourier-Laplace Transform we get,

$$g(t,z,x,y) = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{i\{(st+uz)-i(px+vy)\}} G(s,u,p,v)\, ds\, du\, dp\, dv \quad (4.2)$$

Taking complex conjugate on both sides of (4.2) we get

$$\overline{g(t,z,x,y)} = -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{-i\{(st+uz)-i(px+vy)\}}\overline{G(s,u,p,v)}\, ds\, du\, dp\, dv \quad (4.3)$$

Now by using this equation (4.3) we have,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)\overline{g(t,z,x,y)}\, dt\, dz\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)\, dt\, dz\, dx\, dy \left\{ -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{-i\{(st+uz)-i(px+vy)\}}\overline{G(s,u,p,v)}\, ds\, du\, dp\, dv \right\}$$

$$= -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)\overline{G(s,u,p,v)} e^{-i\{(st+uz)-i(px+vy)\}}\, dt\, dz\, dx\, dy\, ds\, du\, dp\, dv$$

(On changing the order of integration)

$$= -\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\{(st+uz)-i(px+vy)\}} f(t,z,x,y)\overline{G(s,u,p,v)}\, ds\, du\, dp\, dv\, dt\, dz\, dx\, dy$$

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$$=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{G(s,u,p,v)}\,ds\,du\,dp\,dv\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)\,e^{-i\{(st+uz)-i(px+vy)\}}\;dt\,dz\,dx\,dy\right]$$

$$=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\overline{G(s,u,p,v)}\,ds\,du\,dp\,dv\,\{F(s,u,p,v)\}$$

(by definition of two dimensional Fourier-Laplace Transform)

$$=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}F(s,u,p,v)\overline{G(s,u,p,v)}\,ds\,du\,dp\,dv$$

Thus we have proved that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)\overline{g(t,z,x,y)}\,dt\,dz\,dx\,dy$$

$$=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}F(s,u,p,v)\overline{G(s,u,p,v)}\;ds\,du\,dp\,dv \tag{4.4}$$

Putting $g(t,z,x,y)=f(t,z,x,y)$ also $G(s,u,p,v)=F(s,u,p,v)$

and $\overline{G(s,u,p,v)}=\overline{F(s,u,p,v)}$ in equation (4.4), we get

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)\,\overline{f(t,z,x,y)}\,dt\,dz\,dx\,dy$$

$$=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}F(s,u,p,v)\,\overline{F(s,u,p,v)}\;ds\,du\,dp\,dv \qquad \text{or}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}|f(t,z,x,y)|^2\;dt\,dz\,dx\,dy=-\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_{\gamma-i\infty}^{\gamma+i\infty}|F(s,u,p,v)|^2 ds\,du\,dp\,dv$$

5. MODULATION PROPERTY FOR TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM

5.1. If $FL\{f(t,z,x,y)\}=F(s,u,p,v)$ denotes the generalized two dimensional Fourier-Laplace transform of $f(t,z,x,y)$ then

$$FL\{f(t,z,x,y)\cos(at+bz+cx+dy)\}(s,u,p,v)$$

$$=\frac{1}{2}\{F(s-a,\;u-b,\;p-ic\;\;v-id)+F(s+a,\;u+b,\;p+ic\;\;v+id)\}$$

Proof: $FL\{f(t,z,x,y)\}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\;f(t,z,x,y)\,dt\,dz\,dx\,dy$

$\therefore\quad FL\{f(t,z,x,y)\cos(at+bz+cx+dy)\}$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\;f(t,z,x,y)\cos(at+bz+cx+dy)\,dt\,dz\,dx\,dy$$

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$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\{(st+uz)-i(px+vy)\}} f(t,z,x,y)\left[\frac{e^{i(at+bz+cx+dy)}+e^{-i(at+bz+cx+dy)}}{2}\right] dt\,dz\,dx\,dy$$

$$=\frac{1}{2}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{i(at+bz+cx+dy)}e^{-i\{(st+uz)-i(px+vy)\}}\,dt\,dz\,dx\,dy\right.$$

$$\left.+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i(at+bz+cx+dy)}e^{-i\{(st+uz)-i(px+vy)\}}\,dt\,dz\,dx\,dy\right\}$$

$$=\frac{1}{2}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i\{[(s-a)t+(u-b)z]-i[(p-ic)x+(v-id)y]\}}\,dt\,dz\,dx\,dy\right.$$

$$\left.+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i\{[(s+a)t+(u+b)z]-i[(p+ic)x+(v+id)y]\}}\,dt\,dz\,dx\,dy\right\}$$

$$\therefore \quad FL\{f(t,z,x,y)\cos(at+bz+cx+dy)\}(s,u,p,v)$$

$$=\frac{1}{2}\{F(s-a,\ u-b,\ p-ic\ \ v-id)+F(s+a,\ u+b,\ p+ic\ \ v+id)\}$$

5.2. If $FL\{f(t,z,x,y)\}=F(s,u,p,v)$ denotes the generalized two dimensional Fourier-Laplace transform of $f(t,z,x,y)$ then

$$FL\{f(t,z,x,y)\sin(at+bz+cx+dy)\}(s,u,p,v)$$

$$=\frac{1}{2i}\{F(s-a,\ u-b,\ p-ic\ \ v-id)-F(s+a,\ u+b,\ p+ic\ \ v+id)\}$$

Proof: We have

$$FL\{f(t,z,x,y)\sin(at+bz+cx+dy)\}(s,u,p,v)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\{(st+uz)-i(px+vy)\}} f(t,z,x,y)\sin(at+bz+cx+dy)\,dt\,dz\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\{(st+uz)-i(px+vy)\}} f(t,z,x,y)\left[\frac{e^{i(at+bz+cx+dy)}-e^{-i(at+bz+cx+dy)}}{2i}\right] dt\,dz\,dx\,dy$$

$$=\frac{1}{2i}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{i(at+bz+cx+dy)}e^{-i\{(st+uz)-i(px+vy)\}}\,dt\,dz\,dx\,dy\right.$$

$$\left.-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i(at+bz+cx+dy)}e^{-i\{(st+uz)-i(px+vy)\}}\,dt\,dz\,dx\,dy\right\}$$

$$=\frac{1}{2i}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i\{[(s-a)t+(u-b)z]-i[(p-ic)x+(v-id)y]\}}\,dt\,dz\,dx\,dy\right.$$

$$\left.-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)e^{-i\{[(s+a)t+(u+b)z]-i[(p+ic)x+(v+id)y]\}}\,dt\,dz\,dx\,dy\right\}$$

$$\therefore \quad FL\{f(t,z,x,y)\sin(at+bz+cx+dy)\}(s,u,p,v)$$

$$=\frac{1}{2i}\left\{F\left(s-a,\ u-b,\ p-ic\ \ v-id\right)-F\left(s+a,\ u+b,\ p+ic\ \ v+id\right)\right\}$$

5.3. If $FL\{f(t,z,x,y)\}=F(s,u,p,v)$ denotes the generalized two dimensional Fourier-Laplace transform of $f(t,z,x,y)$ then

$$FL\{f(t,z,x,y)\cos at\cdot\cos bz\cdot\cos cx\cdot\cos dy\}(s,u,p,v)$$

$$=\frac{1}{16}\{F(s-a,\ u-b,\ p-ic\ \ v-id)+F(s-a,\ u-b,\ p-ic\ \ v+id)+F(s-a,\ u-b,\ p+ic\ \ v-id)$$
$$+F(s-a,\ u-b,\ p+ic\ \ v+id)+F(s-a,\ u+b,\ p-ic\ \ v-id)+F(s-a,\ u+b,\ p-ic\ \ v+id)$$
$$+F(s-a,\ u+b,\ p+ic\ \ v-id)+F(s-a,\ u+b,\ p+ic\ \ v+id)+F(s+a,\ u-b,\ p-ic\ \ v-id)$$
$$+F(s+a,\ u-b,\ p-ic\ \ v+id)+F(s+a,\ u-b,\ p+ic\ \ v-id)+F(s+a,\ u-b,\ p+ic\ \ v+id)$$
$$+F(s+a,\ u+b,\ p-ic\ \ v-id)+F(s+a,\ u+b,\ p-ic\ \ v+id)$$
$$+F(s+a,\ u+b,\ p+ic\ \ v-id)+F(s+a,\ u+b,\ p+ic\ \ v+id)\}$$

Proof: $FL\{f(t,z,x,y)\}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\ f(t,z,x,y)\,dt\,dz\,dx\,dy$

$$\therefore\quad FL\{f(t,z,x,y)\cos at\cdot\cos bz\cdot\cos cx\cdot\cos dy\}(s,u,p,v)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\ f(t,z,x,y)\cos at\cdot\cos bz\cdot\cos cx\cdot\cos dy\ dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\ f(t,z,x,y)$$
$$\left[\left(e^{iat}e^{ibz}+e^{iat}e^{-ibz}+e^{-iat}e^{ibz}+e^{-iat}e^{-ibz}\right)\left(e^{icx}e^{idy}+e^{icx}e^{-idy}+e^{-icx}e^{idy}+e^{-icx}e^{-idy}\right)\right]dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\{(st+uz)-i(px+vy)\}}\ f(t,z,x,y)\left[e^{iat}e^{ibz}e^{icx}e^{idy}+e^{iat}e^{ibz}e^{icx}e^{-idy}+e^{iat}e^{ibz}e^{-icx}e^{idy}\right.$$
$$+e^{iat}e^{ibz}e^{-icx}e^{-idy}+e^{iat}e^{-ibz}e^{icx}e^{idy}+e^{iat}e^{-ibz}e^{icx}e^{-idy}+e^{iat}e^{-ibz}e^{-icx}e^{idy}+e^{iat}e^{-ibz}e^{-icx}e^{-idy}$$
$$+e^{-iat}e^{ibz}e^{icx}e^{idy}+e^{-iat}e^{ibz}e^{icx}e^{-idy}+e^{-iat}e^{ibz}e^{-icx}e^{idy}+e^{-iat}e^{ibz}e^{-icx}e^{-idy}+e^{-iat}e^{-ibz}e^{icx}e^{idy}$$
$$\left.+e^{-iat}e^{-ibz}e^{icx}e^{-idy}+e^{-iat}e^{-ibz}e^{-icx}e^{idy}+e^{-iat}e^{-ibz}e^{-icx}e^{-idy}\right]dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}\ dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)\right.$$
$$e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$
$$dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}\ dt\,dz\,dx\,dy$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}\ dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)$$

$$e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$f(t,z,x,y)\,dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)$$

$$e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$f(t,z,x,y)\,dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)$$

$$e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$\left.f(t,z,x,y)\,dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(t,z,x,y)e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy\right\}$$

$$=\frac{1}{16}\{F(s-a,\ u-b,\ p-ic\ v-id)+F(s-a,\ u-b,\ p-ic\ v+id)+F(s-a,\ u-b,\ p+ic\ v-id)$$
$$+F(s-a,\ u-b,\ p+ic\ v+id)+F(s-a,\ u+b,\ p-ic\ v-id)+F(s-a,\ u+b,\ p-ic\ v+id)$$
$$+F(s-a,\ u+b,\ p+ic\ v-id)+F(s-a,\ u+b,\ p+ic\ v+id)+F(s+a,\ u-b,\ p-ic\ v-id)$$
$$+F(s+a,\ u-b,\ p-ic\ v+id)+F(s+a,\ u-b,\ p+ic\ v-id)+F(s+a,\ u-b,\ p+ic\ v+id)$$
$$+F(s+a,\ u+b,\ p-ic\ v-id)+F(s+a,\ u+b,\ p-ic\ v+id)$$
$$+F(s+a,\ u+b,\ p+ic\ v-id)+F(s+a,\ u+b,\ p+ic\ v+id)\}$$

5.4. If $FL\{f(t,z,x,y)\}=F(s,u,p,v)$ denotes the generalized two dimensional Fourier-Laplace transform of $f(t,z,x,y)$ then

$$FL\{f(t,z,x,y)\sin at\cdot\sin bz\cdot\sin cx\cdot\sin dy\}(s,u,p,v)$$

$$=\frac{1}{16}\{F(s-a,\ u-b,\ p-ic\ v-id)-F(s-a,\ u-b,\ p-ic\ v+id)-F(s-a,\ u-b,\ p+ic\ v-id)$$
$$+F(s-a,\ u-b,\ p+ic\ v+id)-F(s-a,\ u+b,\ p-ic\ v-id)+F(s-a,\ u+b,\ p-ic\ v+id)$$
$$+F(s-a,\ u+b,\ p+ic\ v-id)-F(s-a,\ u+b,\ p+ic\ v+id)-F(s+a,\ u-b,\ p-ic\ v-id)$$
$$+F(s+a,\ u-b,\ p-ic\ v+id)+F(s+a,\ u-b,\ p+ic\ v-id)-F(s+a,\ u-b,\ p+ic\ v+id)$$
$$+F(s+a,\ u+b,\ p-ic\ v-id)-F(s+a,\ u+b,\ p-ic\ v+id)$$

$$-F\left(s+a,\ u+b,\ p+ic\ \ v-id\right)+F\left(s+a,\ u+b,\ p+ic\ \ v+id\right)\Big\}$$

Proof: $FL\left\{f\left(t,z,x,y\right)\right\}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{(st+uz)-i(px+vy)\right\}}\ f\left(t,z,x,y\right)dt\,dz\,dx\,dy$

$$\therefore\quad FL\left\{f\left(t,z,x,y\right)\sin at\cdot\sin bz\cdot\sin cx\cdot\sin dy\right\}\left(s,u,p,v\right)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{(st+uz)-i(px+vy)\right\}}\ f\left(t,z,x,y\right)\sin at\cdot\sin bz\cdot\sin cx\cdot\sin dy\ dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{(st+uz)-i(px+vy)\right\}}\ f\left(t,z,x,y\right)$$

$$\left[\left(e^{iat}e^{ibz}-e^{iat}e^{-ibz}-e^{-iat}e^{ibz}+e^{-iat}e^{-ibz}\right)\left(e^{icx}e^{idy}-e^{icx}e^{-idy}-e^{-icx}e^{idy}+e^{-icx}e^{-idy}\right)\right]dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{(st+uz)-i(px+vy)\right\}}\ f\left(t,z,x,y\right)\Big[e^{iat}e^{ibz}e^{icx}e^{idy}-e^{iat}e^{ibz}e^{icx}e^{-idy}-e^{iat}e^{ibz}e^{-icx}e^{idy}$$

$$+e^{iat}e^{ibz}e^{-icx}e^{-idy}-e^{iat}e^{-ibz}e^{icx}e^{idy}+e^{iat}e^{-ibz}e^{icx}e^{-idy}+e^{iat}e^{-ibz}e^{-icx}e^{idy}-e^{iat}e^{-ibz}e^{-icx}e^{-idy}$$

$$-e^{-iat}e^{ibz}e^{icx}e^{idy}+e^{-iat}e^{ibz}e^{icx}e^{-idy}+e^{-iat}e^{ibz}e^{-icx}e^{idy}-e^{-iat}e^{ibz}e^{-icx}e^{-idy}+e^{-iat}e^{-ibz}e^{icx}e^{idy}$$

$$-e^{-iat}e^{-ibz}e^{icx}e^{-idy}-e^{-iat}e^{-ibz}e^{-icx}e^{idy}+e^{-iat}e^{-ibz}e^{-icx}e^{-idy}\Big]dt\,dz\,dx\,dy$$

$$=\frac{1}{16}\Bigg\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}\ dt\,dz\,dx\,dy-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)$$

$$e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s-a)t+(u-b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}\ dt\,dz\,dx\,dy$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}\ dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)$$

$$e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$f\left(t,z,x,y\right)dt\,dz\,dx\,dy-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s-a)t+(u+b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}\ dt\,dz\,dx\,dy$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}}\ dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)$$

$$e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}}dt\,dz\,dx\,dy+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$f\left(t,z,x,y\right)dt\,dz\,dx\,dy-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f\left(t,z,x,y\right)e^{-i\left\{\left[(s+a)t+(u-b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}}\ dt\,dz\,dx\,dy$$

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$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y) e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p-ic)x+(v-id)y\right]\right\}} dt\, dz\, dx\, dy - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y)$$

$$e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p-ic)x+(v+id)y\right]\right\}} dt\, dz\, dx\, dy - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p+ic)x+(v-id)y\right]\right\}}$$

$$\left. f(t,z,x,y)\, dt\, dz\, dx\, dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t,z,x,y) e^{-i\left\{\left[(s+a)t+(u+b)z\right]-i\left[(p+ic)x+(v+id)y\right]\right\}} dt\, dz\, dx\, dy \right\}$$

$$=\frac{1}{16}\{F(s-a,\ u-b,\ p-ic\ \ v-id)-F(s-a,\ u-b,\ p-ic\ \ v+id)-F(s-a,\ u-b,\ p+ic\ \ v-id)$$
$$+F(s-a,\ u-b,\ p+ic\ \ v+id)-F(s-a,\ u+b,\ p-ic\ \ v-id)+F(s-a,\ u+b,\ p-ic\ \ v+id)$$
$$+F(s-a,\ u+b,\ p+ic\ \ v-id)-F(s-a,\ u+b,\ p+ic\ \ v+id)-F(s+a,\ u-b,\ p-ic\ \ v-id)$$
$$+F(s+a,\ u-b,\ p-ic\ \ v+id)+F(s+a,\ u-b,\ p+ic\ \ v-id)-F(s+a,\ u-b,\ p+ic\ \ v+id)$$
$$+F(s+a,\ u+b,\ p-ic\ \ v-id)-F(s+a,\ u+b,\ p-ic\ \ v+id)$$
$$-F(s+a,\ u+b,\ p+ic\ \ v-id)+F(s+a,\ u+b,\ p+ic\ \ v+id)\}$$

6. CONCLUSION

In this paper we established a Two Dimensional Fourier-Laplace transform and investigated the Linearity property, Parsevals theorem and Modulation theorem of Two Dimensional Fourier-Laplace transform. The work may be useful for solving higher order ordinary and partial differential equations as well as integral equations.

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